Q.No- verify Lagranges mean value theorem in $[0, \frac{1}{2}]$ for the function $f(x) = x(x-1)(x-2)$.

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Soln - Here, we see that $f(x)$ is continuous over $[0, \frac{1}{2}]$ and differentiable over $]0, \frac{1}{2}[$

Hence, the hypothesis of Lagranges mean value theorem is satisfied

Now, we find a point c in $]0, \frac{1}{2}[$ such that,

$$\frac{f(\frac{1}{2}) - f(0)}{\frac{1}{2} - 0} = f'(c) \quad \text{--- (1)}$$

$\because f(x) = x(x-1)(x-2)$

$f(x) = x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$

$\therefore f'(x) = 3x^2 - 6x + 2$ over $]0, \frac{1}{2}[$.

Hence from (1), we have

$$\frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2) - 0}{\frac{1}{2} - 0} = 3c^2 - 6c + 2$$

$$-\frac{1}{2}\left(-\frac{3}{2}\right) = 3c^2 - 6c + 2$$

$$= \frac{3}{4} = 3c^2 - 6c + 2$$

or, $12c^2 - 24c + 8 = 3$

or, $12c^2 - 24c + 5 = 0$

$$\therefore c = \frac{24 \pm \sqrt{(24)^2 - 4 \times 12 \times 5}}{2 \times 12} \quad \left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$c = \frac{24 \pm \sqrt{24(24-10)}}{24}$$

$$= \frac{24 \pm \sqrt{24 \times 14}}{24}$$

$$= \frac{24 \pm \sqrt{6 \times 4 \times 7 \times 2}}{24}$$

$$= \frac{24 \pm 2\sqrt{3 \times 2 \times 7 \times 2}}{24}$$

$$= \frac{24 \pm 4\sqrt{21}}{24}$$

$$= \frac{\cancel{24}}{\cancel{24}} \pm \frac{\cancel{4}}{\cancel{24}_{6}} \sqrt{21}$$

$$= 1 \pm \frac{1}{6}\sqrt{21}$$

$\because c \in \left]0, \frac{1}{2}\right[$

So, +ve sign is not possible.

Q No- Examine the validity of the hypothesis and the conclusion of Lagranges mean value theorem for the function f defined by $f(x) = |x|$ for every $x \in [2, -1]$

Soln- Here we see that $f(x)$ is continuous over $[-2, 1]$ but $f(x)$ is not differentiable over $]-2, 1[$ since $f(x)$ is not diff at 0.

Hence, the hypothesis of Lagranges mean value theorem is not satisfied for this function.

If there exist a pt. $c \in]-2,1[$ such that,

$$\frac{f(1)-f(-2)}{1-(-2)} = f'(c).$$

or, $\frac{1-2}{1+2} = f'(c)$

or, $f'(c) = -\frac{1}{3}$

$\therefore f(x) = -\frac{1}{3}x + k$ at $x=c$.

This does not tally the function $f(x) = |x|$

Hence, the hypothesis and the conclusions are both false.

QNo.- Verify Cauchy's mean value theom. for the function x^3 and x^4 in $[1,2]$.

Soln Let $f(x) = x^3$

and $g(x) = x^4$.

We see that both function $f(x)$ and $g(x)$ are continuous over $[1,2]$ and differentiable over $]1,2[$.

$2^3 = 8$

$2^4 = 16$

Now, $\frac{f(2)-f(1)}{g(2)-g(1)} = \frac{8-1}{16-1} = \frac{7}{15}$ — (1)

and, $\frac{f'(x)}{g'(x)} = \frac{3x^2}{4x^3} = \frac{3}{4x}$

$\therefore \frac{f'(c)}{g'(c)} = \frac{3}{4c}$ — (2)

But for mean value theo^{m} -

$$\frac{f(2)-f(1)}{g(2)-f(1)} = \frac{f'(c)}{g'(c)}$$

or, $\frac{7}{15} = \frac{3}{4c}$ from (1) & (2)

$$\therefore c = \frac{45}{28}.$$

Here, we see that c lies in $]1,2[$.

Hence Cauchy's mean value theo^{m} is verified.